

18th International Workshop on Matrices and Statistics 2009

June 23 – 27, 2009, Smolenice Castle, Slovak Republic

***18th International Workshop on
Matrices and Statistics 2009***

Program and Abstracts



Smolenice Castle, Slovakia

June 23 – 27, 2009

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Smolenice Castle, Slovak Republic, June 23 – 27, 2009

Editors: Viktor Witkovský and Júlia Volaufová

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General Information

Conference

The 18th International Workshop on Matrices and Statistics, Smolenice Castle, Slovakia, June 23-27, 2009.

Endorsed by the International Linear Algebra Society

This is the next in a long-running series of Workshops with the purpose of stimulating exchanges of ideas and research at the interfaces of matrix theory, statistics, and stochastic processes.

International Organizing Committee

- Júlia Volaufová - Chair (USA)
- Simo Puntanen - Vice-Chair (Finland)
- George P. H. Styan - Honorary Chair (Canada)
- Augustyn Markiewicz - (Poland)
- Jeffrey J. Hunter (New Zealand)
- S. Ejaz Ahmed (Canada)
- Götz Trenkler (Germany)
- Dietrich von Rosen (Sweden)
- Hans Joachim Werner (Germany)

Local Organizing Committee

- Viktor Witkovský - Chair (Slovakia)
- Gejza Wimmer - Vice-Chair (Slovakia)
- Barbora Arendacká (Slovakia)
- Svorad Štolc Jr. (Slovakia)
- Katarina Cimermanová (Slovakia)
- Stanislav Katina (Slovakia)

Workshop Venue

Smolenice Castle - the Congress Centre of the Slovak Academy of Sciences.

Address of the Smolenice Castle:

Congress Centre of the SAS
Zámocká 18,
919 04 Smolenice, Slovak Republic
Phone: +421 33 5586 191-2,
Fax: +421 33 5586 193

Position of the Smolenice Castle:

LATITUDE: 48° 30' 50.05" N
LONGITUDE: 17° 25' 58.10" E
ELEVATION: 293 m



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General Information

Registration and Information Desk

Registration and Information Desk is located in the ground floor of the Smolenice Castle (near the main entrance). The desk will be open during following hours:

June 23	11:30 – 19:00
June 24 – 26	8:00 – 18:00 (with a lunch break)
June 27	8:00 – 13:00

Look for the actual information on the boards at the registration desk and at the entrance to the main lecture room.

Information for Presenters

Oral Presentations

- Oral presentations will take place in the lecture room at the first floor of the Castle.
- The pre-planned time is 40 minutes for an invited lecture and 20 minutes for a contributed presentation (including discussion).
- We recommend to prepare the presentations for an electronic presentation either as an Adobe Acrobat file or as a Microsoft PowerPoint presentation.
- Presenting authors should upload the files (available on CD or USB flash drive) at least 30 minutes before the beginning of the Session.
- Usage of your own computer for presentation is not possible.

Poster presentations

- Posters will be displayed at the lobby in the second floor, near the lecture room.
- The maximum allowable size of the Posters is 120 x 80 cm (length x width).
- The posters will be displayed from Tuesday till Friday.

Meals and Refreshments

Breakfasts, lunches and dinners will be served in the main dining room in the first floor, near the main castle staircase.

Breakfast	8:00 – 9:00
Lunch	12:00 – 13:00 (13:00 – 14:00 on Tuesday, June 23)
Dinner	19:00 – 20:00 (18:00 – 19:00 on Wednesday, June 24)

Refreshments during coffee breaks will be served in the lobby at the main lecture room in the first floor.

Individual refreshments can be purchased in the bar in front of the main dining room (opened each day till 21:30).

Social Programme

A Welcome Reception is planned for Tuesday, June 23 at 18:00-19:00, in Red Hall (Červený salónik) of the Castle.

Excursion to the Castle Červený Kameň is planned for Thursday, June 25 in the afternoon at 13:30.



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Additional social program for the accompanying persons includes Introducing the Smolenice Castle, Visit of traditional pottery producer in Modra, Visit of the Piešťany Spa, Walk through the Castle park and Molpir.

Departure from Smolenice

We kindly ask the participants to free their rooms on Saturday, June 27, till 9:00 (AM).

The conference bus from Smolenice Castle to Bratislava will depart on Saturday, June 27, 2009, shortly after the lunch, approximately at 13:00. The bus will stop at the Bratislava Airport, the Bus Station, and the city center. The journey to Bratislava will take approximately 90 minutes.

Contact Information

18th International Workshop on Matrices and Statistics 2009

Viktor Witkovský

Institute of Measurement Science

Slovak Academy of Sciences

Dúbravská cesta 9

84104 Bratislava, Slovakia

E-mail: witkovsky@savba.sk

Tel.: +421 905 223 191

Fax.: +421 2 54775943 or +421 2 59104587

Web: <http://www.um.sav.sk/en/iwms2009.html>



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Scientific Program

Tuesday, June 23, 2009

9:30 - 11:30 **Conference Bus - Bratislava to Smolenice**

11:30 - 13:00 **Registration**

13:00 - 14:00 **Lunch**

14:20 - 14:40 **Opening of the IWMS 2009**

Session I

Chairman: Ivo Marek

14:40 - 15:20 Miroslav Fiedler (Invited Lecture)
New Properties of Cauchy Matrices

15:20 - 15:40 Frank Jerry Hall
Sign Patterns That Require Almost Unique Rank

15:40 - 16:20 **Coffee Break**

Session II

Chairman: Simo Puntanen

16:20 - 17:00 Karl Gustafson (Invited Lecture)
On My Min-Max Theorem (1968) And Its Consequences

17:00 - 17:20 Peter Maličský
A System of Differential Equations Associated with Two Matrices with Antisymmetric Sum

17:20 - 17:40 Anne Selart
Copula Based on Skew Normal Distribution

18:00 - 19:00 **Welcome Drink**

19:00 - 20:00 **Dinner**



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Scientific Program

Wednesday, June 24, 2009

8:00 - 9:00 **Breakfast**

Session III

Chairman: Miroslav Fiedler

9:00 - 9:40 Ivo Marek (Invited Lecture)
Multi-Level Matrix Computations

9:40 - 10:00 Jeffrey J Hunter
On the Stochastic Properties of Semi-Magic and Magic Markov Chains

10:00 - 10:20 Jolanta Grala-Michalak
Premium Principles Based on Quantiles for the Certain Type of Distribution

10:20 - 11:00 **Coffee Break**

Session IV

Chairman: Roman Zmyślony

11:00 - 11:20 Sandra Saraiva Ferreira, Dário Ferreira, Célia Nunes, João Tiago Mexia
Inference for Almost Balanced Models with Multiple Comparisons

11:20 - 11:40 Dário Ferreira, Sandra Saraiva Ferreira, Célia Nunes, João Tiago Mexia
Maximum Likelihood Estimators for Mixed Normal Models, the Newton-Raphson Method

11:40 - 12:00 Célia Nunes, Sandra Ferreira, Dário Ferreira, João Tiago Mexia
Generalized F Distributions with Non-Centrality Parameters with the Convolution of Gamma and Beta Distributions

12:00 - 13:00 **Lunch**

Session V

Chairman: George Styan

14:00 - 14:40 Andrej Pázman (Invited Lecture)
Matrix Analysis of The Information Content in Experiments with Correlated Observations

14:40 - 15:00 Phil Bertrand
When Can Statistics Lie?

15:00 - 15:20 Radoslav Harman, Thomas Klein
Kiefer Completeness in Quadratic Regression Models on Permutation-Reflection Invariant Experimental Domains

15:20 - 15:40 Märt Möls
Identifiability of Linear Combinations of Random Effects in Linear Mixed Model

15:40 - 16:20 **Coffee Break**



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Wednesday, June 24, 2009

Session VI		
Chairman: Lynn LaMotte		
16:20 - 17:00		Mike Kenward (Invited Lecture) <i>Doubly Robust Multiple Imputation</i>
17:00 - 17:20		Tatjana von Rosen <i>On the Inverse of the Patterned Covariance Matrix in a Balanced Mixed Anova Model</i>
17:20 - 17:40		Martin Ohlson, Dietrich von Rosen <i>Explicit Estimators of Parameters in the Growth Curve Model with Linearly Structured Covariance Matrices</i>
17:40 - 18:00		Dietrich von Rosen <i>Restrictions on the Mean Parameters Via Linear Equations in Multivariate Linear Models</i>
18:00 - 19:00		Dinner
19:00 - 20:00		Poster Session



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Thursday, June 25, 2009

8:00 - 9:00 **Breakfast**

Session VII

Chairman: Mike Kenward

9:00 - 9:40 Friedrich Pukelsheim, Bruno Simeone (Invited Lecture)
An L_1 -analysis of the Iterative Proportional Fitting Procedure

9:40 - 10:00 Natalja Lepik
Estimation with Restrictions in Survey Sampling

10:00 - 10:20 Imbi Traat
Calibration of Estimates on the Information from Other Samples

10:20 - 11:00 **Coffee Break**

Session VIII

Chairman: Friedrich Pukelsheim

11:00 - 11:40 Lubomír Kubáček (Invited Lecture)
Criterion Matrices for a Parameter Elimination

11:40 - 12:00 Eva Fišerová, Karel Hron
Orthogonal Regression for Compositional Data Using a Linear Statistical Model with Type-II Constraints

12:00 - 13:00 **Lunch**

13:30 - 17:00 **Excursion to the Castle "Červený Kameň"**

17:30 - 18:30 **Wine tasting in the Chapel of the Smolenice Castle**

19:00 - 20:00 **Dinner**



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Friday, June 26, 2009

8:00 - 9:00 **Breakfast**

Session IX

Chairman: Jeffrey Hunter

9:00 - 9:40 Yonghui Liu (Invited Lecture)
Simultaneous Decomposition for Matrices with Statistical Applications

9:40 - 10:00 Esra Akdeniz Duran, Fikri Akdeniz
The Efficiency of Modified Jackknifed Liu-Type Estimator

10:00 - 10:20 Jan Hauke
Matrix Completion Problems for Correlation Matrices

10:20 - 11:00 **Coffee Break**

Session X

Chairman: Alastair Scott

11:00 - 11:40 Tõnu Kollo, Dietrich von Rosen, Muni S. Srivastava
Testing Structures of the Dispersion Matrix: A Non-Normal Approach

11:40 - 12:00 Hans Joachim Werner
Controlling the Bias of Linear Predictors

12:00 - 13:00 **Lunch**

Session XI

Chairman: Dietrich von Rosen

14:00 - 14:40 Roman Zmyślony, Jacek Bojarski (Invited Lecture)
An Algorithm for Least Squares Estimation of Parameters in Nonlinear Regression Models

14:40 - 15:00 Simo Puntanen, Stephen J. Haslett
Effect of Adding Regressors on the Equality of the BLUEs Under Two Linear Models

15:00 - 15:20 Lynn Roy LaMotte
Following K. Pearson to Test the General Linear Hypothesis

15:20 - 15:40 Hilmar Drygas
Why Gauss Did Not Discover Gram-Schmidt Orthogonalization

15:40 - 16:20 **Coffee Break**



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Scientific Program

Friday, June 26, 2009

Session XII	
Chairman: Tõnu Kollo	
16:20 - 16:40	George P.H. Styan <i>An Illustrated Introduction to 4 x 4 Latin Squares in Europe: 1283–1788</i>
16:40 - 17:00	Gerald E. Subak-Sharpe, George P.H. Styan <i>Some Comments on Terminal Weight Numbers and the Campbell-Youla Generalized Inverse for Resistive Electrical Networks</i>
17:00 - 17:20	Burkhard Schaffrin <i>TLS-Collocation: The Total Least-Squares Approach to EIV-Models with Prior Information</i>
17:20 - 17:40	Stanislav Katina <i>Thin-Plate Spline Relaxation Reconsidered: From Spatial Data to Shape Data Examples</i>
19:00	Conference Dinner



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Saturday, June 27, 2009

8:00 - 9:00 **Breakfast**

Session XIII

Chairman: Hans Joachim Werner

9:30 - 10:10 Alastair Scott (Invited Lecture)

Semi-Parametric Methods for Regression with Response-Selective Sampling

10:10 - 10:30 František Rublík

On Asymptotic Normality and Testing of Equality of the First h Eigenvectors of Covariance Matrices

10:30 - 10:50 Viktor Witkovský, Martina Chvosteková

On Simultaneous Tolerance Intervals in Linear Regression

10:50 - 11:10 Júlia Volaufová, Lynn Roy LaMotte

Summary Measures and Random Coefficients Models

11:10 - 11:30 **Closing of the IWMS 2009**

11:30 - 12:30 **Lunch**

13:00 - 15:00 **Conference Bus - Smolenice to Bratislava**



Posters

- P1 Fikri Akdeniz, Esra Akdeniz Duran
New Difference-Based Estimator of Parameters in Semiparametric Regression Models
- P2 Barbora Arendacká
Independent Chi-Squares in Linear Mixed Models
- P3 Hana Boháčová, Jana Heckenbergerová
On the Nonsensitiveness Regions in the Normal Mixed Linear Model with Type II Constraints
- P4 Martina Chvosteková, Viktor Witkovský
Exact Likelihood Ratio Test for the Parameters of the Linear Regression Model with Normal Errors
- P5 Katarína Cimermanová
Classification of Noisy Data with an Application to Breath Gas Analysis
- P6 Lukáš Laffers, Marian Grendár
Empirical Likelihood Estimation in Interest Rate Diffusion Models
- P7 Martina Hančová
Special Matrices in Time Series Forecasting
- P8 Jana Heckenbergerová, Hana Boháčová, Jaroslav Marek
Plug-In Problem in Discrete GNSS-PIM Algorithms and Nonsensitiveness Region for Test of Position Integrity
- P9 Klára Hornišová
Calibration Intervals for Values of Concentration Based on Measurements of Voltage
- P10 Gejza Wimmer
Locally Best Linear-Quadratic Unbiased Estimators of the Covariance Matrix Elements in a Special Heteroscedastic Regression Model
- P11 Gejza Wimmer Jr.
Comparison of Different Confidence Regions for Regression Parameter in Linear Mixed Model
- P12 Ivan Žežula, Daniel Klein
Orthogonal Decompositions in Growth Curve Models



**New Properties of Cauchy Matrices
(Invited Lecture)**

Miroslav Fiedler

Institute of Computer Science, Academy of Sciences of the Czech Republic
Prague, Czech Republic

Cauchy matrices form a class of matrices which is relatively broad (if square of order n , it has $2n - 1$ parameters). On the other hand, it can be very well applied since, among others, the inverses - under mild existence conditions - can be, at least theoretically, explicitly determined. We shall present an apparently new characterization of totally positive square Cauchy matrices and some related results.

Sign Patterns That Require Almost Unique Rank

Frank Hall

Georgia State University, Atlanta, USA

A sign pattern matrix is a matrix whose entries come from the set $\{+, -, 0\}$. The minimum rank $mr(A)$ (maximum rank $MR(A)$) of a sign pattern matrix A is the minimum (maximum) of the ranks of the real matrices in the sign pattern class of A . Several results concerning sign patterns A that require almost unique rank, that is to say, the patterns A such that $MR(A) = mr(A) + 1$ are established. In particular, a complete characterization of these sign patterns is given. Further, the results on sign patterns that require almost unique rank are extended to sign patterns A for which the spread is $d = MR(A) - mr(A)$

**On My Min-Max Theorem (1968) And Its Consequences
(Invited Lecture)**

Karl Gustafson

Department of Mathematics, University of Colorado, Boulder, CO, USA

Central to the origins of my operator trigonometry, a theory in which I initiated the concepts of antieigenvalues and antieigenvectors, is my 1968 Min-Max Theorem [1]. I will discuss its motivation, proof, and consequences. Special attention will be given to its applications to matrix theory and more recently, to statistics.

References:

- [1] K. Gustafson, A min-max theorem. Notices American Math. Soc.15 (1968) 799.
- [2] K. Gustafson, The angle of an operator and positive operator products, Bull. Amer. Math. Soc. 74(1968) 488-492.
- [3] K. Gustafson, A note on left multiplication of semigroup generators, Pacific J. Math. 24(1968) 463-465.
- [4] K. Gustafson, Positive (noncommuting) operator products and semigroups, Math. Zeit.105 (1968) 160-172.
- [5] K. Gustafson, Antieigenvalue inequalities in operator theory, Inequalities III (O.Shisha, ed.), Academic Press (1972) 115-119.



- [6] K. Gustafson, Lectures on computational fluid dynamics, mathematical physics, and linear algebra, World-Scientific (1997).
- [7] K. Gustafson, and D. Rao, Numerical range, Springer (1997).
- [8] K. Gustafson, On geometry of statistical efficiency (1999). Preprint.
- [9] K. Gustafson, Operator trigonometry of statistics and econometrics, Linear Alg. & Its Applications 354 (2002) 141-158.
- [10] K. Gustafson, The trigonometry of matrix statistics, International Statistical Review 74 (2006) 187-202.

A System of Differential Equations Associated with Two Matrices with Antisymmetric Sum

Peter Maličský

Matej Bel University, Banská Bystrica, Slovak Republic

We consider a system of differential equations

$$\mathbf{w}'(t) = \mathbf{A}\mathbf{w}(t) + \mathbf{v}(t),$$

$$\mathbf{v}'(t) = \mathbf{B}\mathbf{v}(t),$$

$$\mathbf{v}(0) = \mathbf{v}_0,$$

$$\mathbf{w}(0) = \mathbf{o},$$

where \mathbf{A} , \mathbf{B} are real matrices of the type $n \times n$ and $\mathbf{w}(t)$, $\mathbf{v}(t)$ are functions with values in \mathbb{R}^n . We assume that $\mathbf{A} + \mathbf{B}$ is antisymmetric, i.e.

$$\mathbf{A} + \mathbf{B} = -(\mathbf{A} + \mathbf{B})^T.$$

The question is, whether the initial value \mathbf{v}_0 may be chosen such that

$$\liminf_{t \rightarrow +\infty} \| e^{-\omega t} \mathbf{w}(t) \| > 0,$$

where ω is a maximum of real parts of eigenvalues of matrices \mathbf{A} and \mathbf{B} and $\| \cdot \|$ is an Euclidean norm. We show in which cases the answer is positive; however a general problem is open.

Copula Based on Skew Normal Distribution

Anne Selart

University of Tartu, Estonia

Copula function based on skew normal distribution is examined. Skew normal distribution is obtained from normal distribution by adding one extra parameter to bring in skewness (Azzalini, Dalla Valle, 1996). As copula is a function that carries the dependence structure of distribution, the copula based on skew normal distribution will retain the skewness a property that should make that copula preferable to widely used Gaussian copula in modeling skewed data, for example. Some properties of the skew normal copula are considered and comparison with the Gaussian copula made.

References:

Azzalini, A. and Dalla Valle, A. (1996). The multivariate skew-normal distribution. *Biometrika* 86, 715-726.



**Multi-Level Matrix Computations
(Invited Lecture)**

Ivo Marek

Czech Institute of Technology, School of Civil Engineering, Prague, Czech Republic

Stochastic modeling in quite many areas of Science and Technology requires computation of characteristics of large scale Markov chains. A serious difficulty appears to be a possible cyclicity of the appropriate transition matrices. A natural tool to solve such problems is proposed: It is the Iterative Aggregation/Disaggregation Method. Its power both within theory and practice as well as some relations to the celebrated Google Lemma will be shown.

On the Stochastic Properties of Semi-Magic and Magic Markov Chains

Jeffrey J Hunter

Auckland University of Technology, New Zealand

Gustafson and Styan (“Super-stochastic matrices and Magic Markov chains”, Linear Algebra Applications, (2008), available online) examined the mathematical properties of super-stochastic matrices, the transition matrices of “magic” Markov chains formed from scaled “magic” squares.

This paper explores the stochastic properties of such chains as well as “semi-magic” chains (with doubly-stochastic transition matrices). Stationary distributions, generalized inverses of Markovian kernels, mean first passage times, variances of the first passage times and expected times to mixing are considered. Some general results are developed, some observations from chains generated by MATLAB are discussed, some conjectures are presented and some special cases (involving three and four states) are explored in detail.

**Premium Principles Based on Quantiles for the Certain Type of
Distribution**

Jolanta Grala-Michalak

Adam Mickiewicz University of Poznań, Poland

This presentation shows the different calculation of an insurance premium based on L -statistics for the mixture of independent uniform and degenerate distributions. Some simple examples are considered, too.

Inference for Almost Balanced Models with Multiple Comparisons

Sandra Saraiva Ferreira¹, Dário Ferreira¹, Célia Nunes¹ and João Tiago Mexia²

¹Mathematics Department, University of Beira Interior, Covilhã, Portugal

²Mathematics Department, Faculty of Science and Technology, New University of Lisbon, Monte da Caparica, Portugal



Commutative Jordan Algebras are used to derive F tests both for estimable vectors and variance components in almost balanced models, which are obtained from any balanced model by considering different number of observations for the treatments. A multiple comparisons procedure is presented.

References:

- [1] Benjamini, Y. and Hochberg Y. (1995). Controlling the false discovery rate: a practical and powerful approach to multiple testing. *J.R. Stat. Soc. Ser. B*, 57, 289-300.
- [2] Fisher, R. A. (1935). *The design of experiments*. Oliver and Boyd, Edinburgh.
- [3] Fonseca, M.; Mexia, J. T. and Zmysłony, R. (2006). Binary operations on Jordan algebras and orthogonal normal models. *Linear Algebra and It's Applications*.
- [4] Fonseca, M.; Mexia, J. T. and Zmysłony, R. (2007). Jordan algebras, generating pivot variables and orthogonal normal models. *Journal of Interdisciplinary Mathematics*, Vol. 10, No. 2, 305-326.
- [5] Khuri, A. I.; Matthew, T. and Sinha, B. K. (1997). *Statistical Tests for Mixed Linear Models*. Wiley Series in Probability and Statistics. John Wiley & Sons - New York.
- [6] Mexia, J. T. (1990). Best linear unbiased estimates duality of F tests and the Scheffé multiple comparison method in presence of controlled heteroscedasticity. *Comp. Stat. & Data Analysis*, 10, No. 3.
- [7] Pereira, D. G. and Mexia, J. T. (2002). Multiple comparisons in joint regression analysis with special reference to variety selection. *Sci. Pap. Agri. Uni. Poznan Agric.*, 3, 67-74.
- [8] Pereira, D. G. and Mexia, J. T. (2008). Selection proposal of cultivars of spring barley in the years from 2001 to 2004, using the joint regression analysis. *Plant Breeding*. doi:10.1111/j.1439-0523.2008.01509.x.
- [9] Saraiva, S. F. (2006). Inference for orthogonal models with segregation. *PhD Thesis*, UBI - Covilhã.
- [10] Stuart, A.; Ord J. K. and Arnold, S. (1999). *Kendall's Advanced Theory of Statistics*. Vol. 88, 597-605.

**Maximum Likelihood Estimators for Mixed Normal Models,
the Newton-Raphson Method**

Dário Ferreira¹, Sandra Saraiva Ferreira¹, Célia Nunes¹ and João Tiago Mexia²

¹Mathematics Department, University of Beira Interior, Covilhã, Portugal

²Mathematics Department, Faculty of Science and Technology, New University of
Lisbon, Monte da Caparica, Portugal

The technique of triple minimization reduces, see Fonseca (2007), the problem of estimation of the parameters of normal mixed models to the estimation of the relative variance components $\gamma_j = \sigma_j^2 / \|\sigma\|^2$, $j = 1, \dots, k$. In this paper, the Newton-Raphson method is used to improve variance component estimators in normal mixed models, which were previously obtained through the technique of triple minimization.

References:

- [1] Ferreira, D., Ferreira, S., Nunes, C. and Mexia, J. T. (2008). Adjustment of normal mixed models through triple minimization.
- [2] Fonseca, M., Carvalho, M., Oliveira, M. and Mexia, J. T. (2007). A Reduction technique for conducting inference in mixed models. *LINSTAT 2008*.
- [3] Fonseca, M. and Mexia, J. T. (2007). Inference in Mixed Models with Stochastic Search.
- [4] Patterson, D. and Thompson, R. (1974). Maximum likelihood estimation of components of variance. *Proceedings of the 8th International Biometric Conference*, 197–207.
- [5] Schott, J. R. (1997). *Matrix Analysis for Statistics*. John Wiley & Sons, Inc, New York.



- [6] Spall, J. C. (2003). *Introduction to Stochastic Search and Optimization. Estimation, Simulation, and Control*, Wiley.
- [7] Searle, S. R., Casella, G. and McCulloch, C. E. (1992). *Variance Components*, Wiley.

**Generalized F Distributions with Non-Centrality Parameters
with the Convolution of Gamma and Beta Distributions**

Célia Nunes¹, Sandra Ferreira¹, Dário Ferreira¹, and João Tiago Mexia²

¹Mathematics Department, University of Beira Interior, Covilhã, Portugal

²Mathematics Department, Faculty of Science and Technology, New University of
Lisbon, Monte da Caparica, Portugal

The quotient of two linear combinations of independent chi-squares has a generalized F distribution. Exact expressions for these distributions, when the chi-squares are central and those in the numerator or in the denominator have even degrees of freedom, were given in Fonseca et al (2002). These expressions were extended for non-central chi-squares with random non-centrality parameters in Nunes and Mexia (2006). This result was also extended when the non-centrality parameters have Gamma distribution, see Nunes et al (2008).

Now we will consider that the random non-centrality parameters are the convolution of a Gamma and a Beta distribution for the usual F distribution and for the generalized F distribution.

References:

- [1] Fonseca, M.; Mexia, J. T. and Zmyślony, R. (2002). Exact distribution for the generalized F tests. *Discussiones Mathematicae, Probability and Statistics*. Vol. 22, No. 1, 2.
- [2] Michalski, A. and Zmyślony, R. (1996). Testing hypothesis for variance components in mixed linear models. *Statistics*, 27, 297–310.
- [3] Michalski, A. and Zmyślony, R. (1999). Testing hypothesis for linear functions of parameters in mixed linear models. *Tatra Mountains Mathematical Publications*, 17, 103–110.
- [4] Nunes, C. and Mexia, J. T. (2006). Non-central generalized F distributions. *Discussiones Mathematicae, Probability and Statistics*. Vol. 26, 47-61.
- [5] Nunes, C., Ferreira, S., Ferreira, D. and Mexia, J. T. (2008). Non-central generalized F distributions with non-centrality parameters with Gamma distribution. *Statistics*. Submitted in October/2008.
- [6] Robbins, H. (1948). Mixture of distribution. *The Annals of Mathematical Statistics*. Vol. 19, 360-369.
- [7] Robbins, H. and Pitman, E.J.G. (1949). Application of the method of mixtures to quadratic forms in normal variates. *The Annals of Mathematical Statistics*. Vol. 20, 552-560.

**Matrix Analysis of The Information Content in Experiments with
Correlated Observations**

(Invited Lecture)

Andrej Pázman

Comenius University, Bratislava, Slovakia

A random field with parametrized mean and parametrized covariance is observed on a finite number of points (= the design). The information obtained from observations is measured via



the Fisher information matrix or via some information functional. If we delete one point from the considered design, or we suppress the influence of this point by adding a "virtual" white noise, then the resulting change of the information matrix (or of the information functional) express the information content of the observation at this point. We use matrix algebra to express this content explicitly, and we give necessary and sufficient conditions to detect design points giving zero information. The presented results are related to some published algorithms for the search of optimal designs for parameters of the mean, and we show that the algebraic structure of the information matrix indicates an analogy between the optimization problem for the estimation of the mean, and the optimization problem for the estimation of the covariance.

Related papers of the author:

- [1] W.G. Müller, A. Pazman (2003): Measures for designs in experiments with correlated errors. *Biometrika* 90, 423-434.
- [2] Pazman (2007): Criteria for optimal design of small-sample experiments with correlated observations. *Kybernetika* 43, 453-462.

When Can Statistics Lie?

Phil Bertrand

Consultant Statistician, Solihull, UK

Simply said: it is

1. When statistical quirks or anti quirks can occur.
2. When data are collected without an experimental design.
3. When there are many correlations between the variables which may affect the explained variable which could be explained, perhaps, by several or a large number of possible explanatory variables.

Can we put together the mathematics and the matrix theory which demonstrates all this?

Kiefer Completeness in Quadratic Regression Models on Permutation-Reflection Invariant Experimental Domains

Radoslav Harman and Thomas Klein*

Comenius University Bratislava, Slovakia
and Technische Universität München, Germany

We consider a saturated second-degree regression model on the I -cube $[-1, 1]^I$, where I is the finite set of factors entering the model. The I -cube's natural symmetry group is generated by permutations of coordinates and reflexions along the coordinate axes, and we analyze moments of designs on the I -cube that are invariant with respect to this group. Based on this analysis we show how invariant designs may be improved relative to the Loewner ordering of moment matrices, and we use this result to construct a Kiefer-complete class of designs. Finally we extend our results to alternative experimental domains with the same symmetry group as the I -cube.



Identifiability of Linear Combinations of Random Effects in Linear Mixed Model

Märt Möls

University of Tartu, Estonia

To derive unique estimates for fixed effects one usually has to use additional reparameterization constraints (or, alternatively, one should consider only estimable linear functions of parameters). Random effects are often (mistakenly) considered to be free of such indeterminacy problems. However, situation can change rapidly, if one does not know exactly the random effects' covariance matrix. Some results concerning identifiability of linear combinations of random effects (for unknown covariance matrix of random effects) will be presented. The usage of reparameterization constraints instead of fixing random effects' covariance structure will be also discussed.

Doubly Robust Multiple Imputation (Invited Lecture)

Mike Kenward

London School of Hygiene & Tropical Medicine, University of London, UK

Missing data are common wherever statistical methods are applied in practice. They present a problem in that they require that additional assumptions be made about the mechanism leading to the incompleteness of the data. By incorporating two models for the missing data process, doubly robust (DR) weighting-based methods offer some protection against misspecification bias since inferences are valid when at least one of the two models is correctly specified. The balance between robustness, efficiency and analytical complexity is one which is difficult to strike, resulting in a split between the likelihood and multiple imputation (MI) school on one hand and the weighting and DR school on the other. A new method is proposed here: doubly robust multiple imputation, which combines the convenience of MI with the robustness of the DR approach. This has an added advantage that we are able to construct approximately DR estimators in settings (such as non-monotone missing at random data) where, hitherto, estimators with this property have not been implemented.

On the Inverse of the Patterned Covariance Matrix in a Balanced Mixed Anova Model

Tatjana von Rosen

Department of Statistics, Stockholm University, Sweden

For a general balanced mixed ANOVA model, Searle and Henderson (1979) studied properties of the covariance matrix of the form $V_s = \sum_{i=0}^1 \theta_i (J_s^{i_s} \otimes J_s^{i_s} \otimes \dots \otimes J_s^{i_s})$, where i is a multipartite number, some of the θ_i equal σ^2 and others are zero. They obtained results concerning the spectrum, determinant and the inverse of such a covariance matrix. The coefficients of such a linear combination are given nonexplicitly. Jiang (2004) gave explicit expressions for these coefficients for V_s^{-1} . Nahtman (2006) extended the results of Searle and



Henderson (1979) concerning the presentation of V_s and its eigenvalues. In this work, some new results regarding the inverse V_s^{-1} will be presented.

Explicit Estimators of Parameters in the Growth Curve Model with Linearly Structured Covariance Matrices

Martin Ohlson^{(a)*} and Dietrich von Rosen^(b)

(a) Department of Mathematics, Linköping University, Sweden

(b) Department of Energy & Technology, Swedish University of Agricultural Sciences, Uppsala, Sweden

Estimation of parameters in the classical Growth Curve model when the covariance matrix has some specific linear structure is considered. In our examples maximum likelihood estimators can not be obtained explicitly and must rely on optimization algorithms. Therefore explicit estimators are obtained as alternatives to the maximum likelihood estimators. From a discussion about residuals, a simple non-iterative estimation procedure is suggested which gives explicit and consistent estimators of both the mean and the linear structured covariance matrix.

Restrictions on the Mean Parameters Via Linear Equations in Multivariate Linear Models

Dietrich von Rosen

Department of Energy & Technology, Swedish University of Agricultural Sciences, Uppsala, Sweden

When testing hypothesis one often puts linear restrictions on the parameters. It is convenient to look upon these restrictions as linear equations. By solving these equations the model may be reparameterized. In a model with mean structure $E[\mathbf{X}] = \mathbf{ABC}$ where \mathbf{A} and \mathbf{C} are known matrices and \mathbf{B} is an unknown parameter matrix the following restrictions will be considered: $\mathbf{GBH} = \mathbf{0}$; $\mathbf{GB} = \mathbf{0}$, $\mathbf{BH} = \mathbf{0}$; $\mathbf{G}_1\mathbf{BH}_1 = \mathbf{0}$, $\mathbf{G}_2\mathbf{BH}_2 = \mathbf{0}$; $\mathbf{G}_1\mathbf{BH}_1 + \mathbf{G}_2\mathbf{\Theta}\mathbf{H}_2 = \mathbf{0}$ as well as some others, where in the expressions the \mathbf{G}_s and \mathbf{H}_s are known matrices and $\mathbf{\Theta}$ is an unknown matrix.

**An L_1 -Analysis of the Iterative Proportional Fitting Procedure
(Invited Lecture)**

Friedrich Pukelsheim* and Bruno Simeone

Universität Augsburg and Sapienza Università di Roma

A new analysis of the Iterative Proportional Fitting procedure is presented. The input data consist of a nonnegative matrix, and of row and column marginals. The output sought is a biproportional fit, that is, a scaling of the input matrix by means of row and column divisors so that the scaled matrix has row and column sums equal to the input marginals. The IPF procedure is an algorithm alternating between the fitting of rows and columns. The structure



of its accumulation points is explored in detail. The progress of the algorithm is evaluated through an L_1 -error function measuring the deviation of current row and column sums from target marginals. A formula is obtained, of max-flow min-cut type, to calculate the minimum L_1 -error directly from the input data. If the minimum L_1 -error is zero, the IPF procedure converges to the unique biproportional fit. Otherwise, it eventually oscillates between various accumulation points.

Estimation with Restrictions in Survey Sampling

Natalja Lepik

University of Tartu, Estonia

Nowadays, the users of official statistics often require that estimates satisfy some certain restrictions. For example in the domain's case this requirement is that the estimators of the domain totals sum up to the population total or to its estimate. Another example is that quarterly estimates have to sum up to the yearly total. It is natural that such relationships are hold for the true population parameters, so they can be considered and used as a kind of the auxiliary information. Involving this information into the estimation process can improve the estimates.

One solution to the described situation is the general restriction (GR) estimator proposed by Knottnerus (2003) that is based on the unbiased initial estimators. The advantages of this GR estimator are the variance minimizing property among other linear estimators satisfying the same restrictions and using the same initial estimators in its construction. But it is well known that there are very many good estimators that are unbiased only asymptotically. We will consider the biased initial estimators, and will construct the new restriction estimator.

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Calibration of Estimates on the Information from Other Samples

Imbi Traat

Institute of Mathematical Statistics, University of Tartu, Estonia

Nowadays, there is often the case that in a present sample survey (PRS) some variables are common with another survey, called the reference survey (RFS), both concerning the same population. It is desired that estimates of the common variables are consistent with each other, e.g. the subpopulation totals estimated in the PRS are additive with the population total estimated in the RFS. The extended calibration approach using known reference totals and estimated in the RFS common variable totals helps here. The matrix tools for inverting block matrices allow getting nice analytical form for the new calibrated estimator that achieves the desired consistency.



**Criterion Matrices For a Parameter Elimination
(Invited Lecture)**

Lubomír Kubáček

Department of Mathematical Analysis and Applications of Mathematics, Palacky University,
Olomouc, Czech Republic

Models with large number of parameters are difficult to deal with. Sometimes an essential part of parameters cannot be even properly interpreted. A problem arises whether some parameters in a model can be neglected. If they are neglected a bias occur in estimators of useful parameters, however the covariance matrix of them is smaller in Loewner sense than the covariance matrix in the true model.

Thus a neighborhood of the origin in the parameter space of neglected parameters can be found, which ensures that the estimators of useful parameters in underparametrized models have smaller mean square errors than the dispersions of the unbiased estimators in the true model. The aim is to find such neighborhoods in different linear regression models.

**Orthogonal Regression for Compositional Data Using a Linear Statistical
Model with Type-II Constraints**

Eva Fišerová and Karel Hron*

Department of Mathematical Analysis and Applications of Mathematics, Palacky University,
Olomouc, Czech Republic

The restrictive properties of compositional data, i.e. multivariate data with positive parts that carry only relative information in their components [1], call for special care to be taken while performing standard statistical methods, e.g. regression analysis. Among the special methods suitable for handling this problem is the total least squares procedure (TLS, orthogonal regression, regression with errors in variables, calibration problem), performed after an appropriate logratio transformation. The difficulty or even impossibility of deeper statistical analysis (confidence regions, hypotheses testing) using the standard TLS techniques can be overcome by calibration solution based on linear statistical models, namely models with type-II constraints (constraints involve in addition to the unknown model's parameters the other unobservable ones), see e.g. [2], [3].

This approach can be combined with standard statistical inference, e.g. confidence and prediction regions and bounds, hypotheses testing, etc., suitable for interpretation of results. Here We deal with the simplest TLS problem where we assume a linear relationship between two errorless measurements of the same object (substance, quantity). We propose an iterative algorithm for estimating the calibration line and also give confidence ellipses for the location of unknown errorless results of measurement. It will be shown that the iterative algorithm converges to the same values as those obtained using the standard TLS techniques. Moreover, illustrative examples from the fields of geology, geochemistry and medicine will be presented and fitted lines and confidence regions interpreted for both original and transformed compositional data.



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**Simultaneous Decomposition for Matrices with Statistical Applications
(Invited Lecture)**

Yonghui Liu

Department of Applied Mathematics, Shanghai Finance University,
Shanghai, P.R. China

Simultaneous decompositions of two or more given matrices can be used to manipulate various matrix operations. One of the most important applications of the simultaneous decompositions is to construct some canonical forms for certain matrix operations of the given matrices. Note only does it provide powerful tools for theoretical analysis, but it allow a method for numerical computation. In this talk we present several simultaneous decompositions of matrices, such as a simultaneous decomposition of matrix pairs (A, B) , $A^* = A$ and a simultaneous decomposition of a matrix triplet (A, B, C) , $A^* = A$. Through the new simultaneous decompositions of matrices, we prove several conjectures on the extremal ranks of matrix pencil $A - BX - (BX)^*$ and $A - BXB^* - CYC^*$. As statistical applications, we provide a new computational method for $BLUE(X\beta)$. We also give some new characterizations of Rao's structure when $BLUE(X\beta) = OLSE(X\beta)$.

The Efficiency of Modified Jackknifed Liu-Type Estimator

Esra Akdeniz Duran^{(a)*} and Fikri Akdeniz^(b)

(a) Department of Statistics, Gazi University, Turkey

(b) Department of Statistics, Çukurova University, Turkey

In this paper, we proposed a new estimator namely, modified jackknifed generalized Liu-type estimator (MJGLE). It is based on the criterion that it combines the ideas underlying both the generalized Liu estimator (GLE) and the jackknifed generalized Liu estimator (JGLE). The performance of MJGLE is compared to that of the GLE and JGLE. The ideas in the paper are illustrated and evaluated using simulations.

Matrix Completion Problems for Correlation Matrices

Jan Hauke

Adam Mickiewicz University, Poznań, Poland

Matrix completion problems are concerned with determining whether partially specified matrices can be completed to fully specified matrices satisfying certain prescribed properties. In the presentation we are interested in these problems for correlation matrices. The problems



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coincide with completion problems connected with nonnegativity definiteness of a matrix A with entries a_{ij} such that

$$a_{ii} = 1, \text{ and } |a_{ij}| \leq 1 \quad (1)$$

Let matrix A be a block matrix not fully defined. For a given matrix A we analyze possible completing of unspecified blocks of the matrix A in such a way that

$$A \geq 0$$

together with (1) are fulfilled.

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Testing Structures of the Dispersion Matrix: A Non-Normal Approach (Invited Lecture)

Tõnu Kollo^{(a)*}, Dietrich von Rosen^(b), Muni S. Srivastava^(c)

^(a)University of Tartu, ^(b)Swedish University of Agricultural Sciences, ^(c)University of Toronto

Three null hypotheses about the covariance structure are examined including sphericity and uncorrelatedness tests in non-normal situation. Used test-statistic for sphericity is based on trace functions for possible applications in high-dimensional set-up. To test uncorrelatedness an asymptotic chi-square statistic is designed. A dependence structure introduced by a linear transformation is examined and asymptotic normal distribution of the sample covariance matrix derived under this assumption. Test statistics for the sphericity and uncorrelatedness tests are designed and their asymptotic distributions from series expansions found in general set-up as well as for some special cases.

Controlling the Bias of Linear Predictors

Hans Joachim Werner

Department of Statistics, Faculty of Economics, University of Bonn, Germany

In the framework of the general (possibly singular) Gauss-Markov model, we are particularly interested in controlling the bias of linear predictors. Some relations between the prediction techniques BLIMBIP and BLUP are also discussed.



**An Algorithm for Least Squares Estimation of Parameters in Nonlinear Regression Models
(Invited Lecture)**

Roman Zmyślony* and Jacek Bojarski

Faculty of Mathematics, Computer Science and Econometrics, University of Zielona Góra,
Poland

The Gauss-Newton and Lavenberg-Marguardt methods are the most popular in the problem of parameters estimation in nonlinear models. These methods are based on the second order Taylor polynomials of the quadratic loss function. G-N method reply Hessian by Jacobian, while L-M method modifies Jacobian in the case when Hessian is not non negative defined. In the paper full information from Hessian is taken to define steps and directions in numerical procedure. Some simulations for regression functions of exponential and Tornguist type function are given.

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Effect of Adding Regressors on the Equality of the BLUEs Under Two Linear Models

Simo Puntanen* and Stephen J. Haslett

Department of Mathematics and Statistics, University of Tampere, Finland
and Institute of Fundamental Sciences, Massey University, Palmerston North, New Zealand

In this talk we consider the estimation of regression coefficients in two partitioned linear models, shortly denoted as $M_{12} = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2, \mathbf{V}\}$ and $\underline{M}_{12} = \{\mathbf{y}, \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2, \underline{\mathbf{V}}\}$, which differ only in their covariance matrices. We call M_{12} and \underline{M}_{12} full models, and correspondingly, $M_i = \{\mathbf{y}, \mathbf{X}_i\boldsymbol{\beta}_i, \mathbf{V}\}$ and $\underline{M}_i = \{\mathbf{y}, \mathbf{X}_i\boldsymbol{\beta}_i, \underline{\mathbf{V}}\}$ small models. We give a necessary and sufficient condition for the equality between the best linear unbiased estimators (BLUEs) of $\mathbf{X}_1\boldsymbol{\beta}_1$ under M_{12} and \underline{M}_{12} . In particular, we consider the equality of the BLUEs under the full models assuming that they are equal under the small models.



Testing the General Linear Hypothesis via K. Pearson's Chi-Squared Statistic

Lynn Roy LaMotte

Biostatistics Program, LSU Health Sciences Center School of Public Health, New Orleans, USA

In a linear model $Y \sim (X\beta, \sigma^2 I)$, powers of tests of $H_0 : H'X\beta = 0$ are developed following Pearson's (1900) formulation. The class considered comprises all tests based on linear statistics $A'Y$ that have expected value 0 under H_0 . The standard F -statistic, which is in this class, has good power properties, but others may be preferred in some settings.

Why Gauss Did Not Discover Gram-Schmidt Orthogonalization

Hilmar Drygas

University of Kassel, Germany

Carl Friedrich Gauss was a user and perhaps also an inventor of the method of Least Squares which he used in astronomy. At first we consider the linear model $y(i) = a + bx(i) + e(i)$ under the usual assumptions on the disturbance. By a very simple method using Steiner's theorem the well-known least squares-estimators of a and b can be easily be determined. This method can readily be generalized to the case of an arbitrary linear model. This method leads unavoidably to the appearances of the Gram-Schmidt orthogonalizers of the regressors. Why did Gauss not discover this method? It was too simple for him.

An Illustrated Introduction to 4×4 Latin Squares in Europe: 1283–1788

George P. H. Styan

Department of Mathematics and Statistics, McGill University, Montréal (Québec), Canada

We identify 4×4 Latin squares which were used in Europe from 1283–1788, and illustrate our findings with postage stamps. In Europe the first appearance of a Latin square must surely be in the set of four 4×4 arrays involving fire, air, water, and land, which comprise the "First Elemental Figure" in the *Ars Demonstrativa* by Ramon Llull (1232–1316), apparently first published in 1283. We have not found any other 4×4 Latin squares used after this in Europe until the 18th century. Published in 1704 in Brussels was what seems to be the first Latin square with numbers — in the book on magic squares by [Abbé François-Guillaume] Poignard, *Grand Chanoine de Bruxelles*. In 1708 an epitaph to Hannibal Bassett (1687–1708) in Cornwall, England, used the four words "all, die, shall, we" set up as a 4-line poem forming a 4×4 Latin square. Two special 4×4 Latin squares may be used to solve the famous Magic Card Puzzle involving the 16 court cards in a regular deck of 52 playing cards; the first solution seems to be by Jacques Ozanam (1640–1717) and Martin Grandin, published in 1723 - 1725. The first application of a 4×4 Latin square in a statistical experimental design seems to be in the study of the feeding of 16 sheep in winter by the French agronomist François Cretté de Palluel (1741–1798), first published in Paris in 1788, just before the French Revolution (1789–1799).



Some Comments on Terminal Weight Numbers and the Campbell-Youla Generalized Inverse for Resistive Electrical Networks

Gerald E. Subak-Sharpe^{(a)*} and George P. H. Styan^(b)

(a) Department of Electrical Engineering, City College of New York, USA

(b) Department of Mathematics and Statistics, McGill University, Montréal (Québec), Canada

We review the most important results in a recent series of papers by the authors concerning resistive electrical networks. These results used metric geometry, graph theory and matrix theory. We concentrate on the Campbell–Youla generalized inverse of the admittance matrix as introduced by the authors [*Linear Algebra and its Applications*, 250, 349–370 (1997)], and on the associated vector w of terminal weight numbers and the network diameter D .

In this paper we derive a curious result, namely the existence of an invariant voltage vector v^* associated with every resistive network. We show that $v^*w = 0$. We also consider the old and largely unsolved problem concerning the impedance matrix of a resistive n -port. We show that terminal weight numbers also represent n -port structure and that for large values of n many different port structures are possible. We concentrate on two extreme port structures: the star-port and the linear-port. We show that for both of these extreme port structures, the sum of the trace elements of the n -port matrices equals D^2 ohms and so equals the square of the network diameter of the $(n + 1)$ -terminal Campbell-Youla resistive embedding matrix.

TLS-Collocation: The Total Least-Squares Approach to EIV-Models with Prior Information

Burkhard Schaffrin

The Ohio State University, Columbus, Ohio, USA

Compared with the standard Gauss-Markov Model, the Errors-In-Variables (EIV) Model gives up the certainty on the entries of the coefficient matrix, but treats the parameter vector as fully unknown. In the presence of prior information, however, these parameters become random effects with known expectation and variance-covariance matrix.

Here, an attempt will be made to predict this random effects vector in a model where the (quasi-linear) relationship to the observation vector is uncertain as well. The procedure is based on an extended least-squares principle and will be called "Total Least-Squares (or TLS) Collocation".



**Thin-Plate Spline Relaxation Reconsidered:
From Spatial Data to Shape Data Examples**

Stanislav Katina^(a,b,c)

(a) Department of Applied Mathematics and Statistics, Comenius University,
Bratislava, Slovak Republic

(b) Neurospin, Institut d'Imagerie BioMédicale, Commissariat à l'Energie Atomique,
Gif sur Yvette, France

(c) Institute of Normal and Pathological Physiology, Slovak Academy of Sciences,
Bratislava, Slovak Republic

In the presentation we discuss thin-plate spline models of the form

$$y_j = f_\lambda(\mathbf{x}_j) + \varepsilon_j, \text{ and}$$

$$\mathbf{y}_j = \mathbf{f}_\lambda(\mathbf{x}_j) + \varepsilon_j, \quad j = 1, 2, \dots, k,$$

where $\mathbf{x}_j = (x_j^{(1)}, x_j^{(2)})^T \in \mathbb{R}^2$, $y_j \in \mathbb{R}$, $\mathbf{y}_j = (y_j^{(1)}, y_j^{(2)})^T \in \mathbb{R}^2$, $\lambda \in \mathbb{R}^+$ is regularization parameter and ε_j are errors ($f : \mathbb{R}^2 \rightarrow \mathbb{R}$ and $\mathbf{f} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$). We apply mentioned methods to the electroencephalogram data - the values of spectral density of external photo-stimulation of 28 subjects in the range 40 Hz from 19 head locations, to the shape data - 22 cephalometric points (landmarks) from digitally processed lateral X-ray films of 48 boys who have complete unilateral cleft of the lip and palate, and 24 human mandibular surfaces with 21 digitized semilandmarks on the symphysis. As it is seen in the examples, we use the methods in the surface (represented by points y_j) flattening and shape (represented by (semi)landmarks \mathbf{y}_j) smoothing by minimizing weighted sums of two energies, pictorial and bending energy, in the relaxation of along homologous curves to get geometrically homologous points, and also in relaxation of multivariate outliers.

**Semi-Parametric Methods for Regression with Response-Selective
Sampling
(Invited Lecture)**

Alastair Scott

Department of Statistics, University of Auckland, Auckland, New Zealand

Suppose that data $\{y_i, x_i, i = 1, \dots, n\}$ are generated from a model with joint density $f(y|x; \beta)g(x)$, where y is a response and x is a vector of covariates. We are interested in estimating the regression parameter, β , when some components of x are missing for some units and where the probability of being missing depends on the value of the response, y , as well as on the remaining, fully-observed, components of x . Values may be missing by accident, as with non-response in survey data, or by design, as in a stratified case-control study where more expensive covariates are only measured on a subset of the experimental units. We are particularly interested in case-control studies with missing data where both types of mechanisms are involved.



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Unlike the situation with ordinary (fully-observed) regression, the full likelihood depends not only on the regression parameters, β , but also on the unknown covariate distribution $g(x)$. We certainly do not want to have to model this covariate distribution in general, so we look for methods that avoid the need for such modeling. We derive the asymptotic covariance matrix of the estimate of β for several such methods, firstly in situations where the probability of response, given the observed data, is known, and secondly in situations where we have to fit a model for the probability. It turns out that the covariance matrix is always smaller in the second situation than in the first. This means that, somewhat counter-intuitively, it is better to estimate the probability that data will be missing, even when that probability is known.

On Asymptotic Normality and Testing of Equality of the First h Eigenvectors of Covariance Matrices

František Rublík

Institute of Measurement Science, Slovak Academy of Sciences, Bratislava, Slovakia

A multisample test of equality of the first h eigenvectors of the covariance matrices of q populations, not requiring normality of the sampled distributions, is presented. The starting point in this scheme is the asymptotic normality of the sample eigenvectors. In the setting when the first h eigenvalues are simple and the sampled distribution need not to be Gaussian, a new condition for this asymptotic normality is presented, and an explicit formula for the column space and the rank of the asymptotic covariance matrix of the sample eigenvectors is given.

On Simultaneous Tolerance Intervals in Linear Regression

Viktor Witkovský* and Martina Chvosteková

Institute of Measurement Science, Slovak Academy of Sciences, Bratislava, Slovak Republic

We consider the problem of constructing the simultaneous tolerance intervals for linear regression. The proposed method follows the approach suggested in Limam and Thomas (1988), however, it is based on inverting the exact likelihood ratio test (LRT) for testing the simple null hypothesis on all parameters of the linear regression model with normally distributed errors, as proposed in Chvosteková and Witkovský (2009).

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Summary Measures and Random Coefficients Models

Júlia Volaufová* and Lynn Roy LaMotte

LSU Health Sciences Center, New Orleans, USA

In many experiments one or more variables are observed repeatedly, mostly over certain time intervals, on independent sampling units (subjects), which are randomly assigned into several treatment groups. The research question sometimes is, whether the relationship between the response variable(s) and time or between the different response variables differs among treatment groups. This setting is routinely modeled by a random coefficients model and the techniques of (linear) mixed models are applied to address the primary aim. An alternative approach is, for each subject to obtain a *summary measure* vector, which characterizes the relationship in question, and test for equality of means of that random vector among treatment groups. Here we study the relationship between the two approaches in some simple settings.

New Difference-Based Estimator of Parameters in Semiparametric Regression Models

Fikri Akdeniz^(a) and Esra Akdeniz Duran^(b)

(a) Department of Statistics, Çukurova University, Turkey

(b) Department of Statistics, Gazi University, Turkey

The paper introduces a new difference-based Liu estimator $\hat{\beta}_{Ldiff} = (\tilde{X}'\tilde{X} + I)^{-1} \times (\tilde{X}'\tilde{y} + \eta\hat{\beta}_{diff})$ of the regression parameters β in the semiparametric regression model, $y = X\beta + f + \varepsilon$. Difference estimators, $\hat{\beta}_{diff} = (\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{y}$, [1], and difference-based Liu estimators are analyzed and compared in the sense of mean-squared error criterion. Finally, the performance of the new estimator is evaluated for a real data set and a Monte Carlo simulation study.

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Independent Chi-Squares in Linear Mixed Models

Barbora Arendacká

Institute of Measurement Science, Slovak Academy of Sciences, Slovak Republic

Burch (2007) suggested confidence intervals for variance components in special cases of unbalanced linear mixed models, i.e. models in which the associated Jordan algebra is non-commutative. He proposed certain independent chi-squared distributed statistics and then employed the generalized inference method. We restate these statistics in terms of a sequence



of projections, in hope to better understand the approach - its limitations and possibilities for extension.

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On the Nonsensitiveness Regions in the Normal Mixed Linear Model with Type II Constraints

Hana Boháčová^(a) and Jana Heckenbergerová^(b)

(a) Institute of Mathematics, Faculty of Economics and Administration, University of Pardubice

(b) Department of Information Technology, Faculty of Electrical Engineering and Informatics, University of Pardubice, Czech Republic

The contributed paper is focused on the estimation of the parameters in the normal mixed linear model with type II constraints (i.e. model of incomplete measurement with conditions) and on the nonsensitiveness regions for the variance of a linear function of the first order parameters.

Exact Likelihood Ratio Test for the Parameters of the Linear Regression Model with Normal Errors

Martina Chvosteková and Viktor Witkovský

Institute of Measurement Science, Slovak Academy of Sciences, Bratislava, Slovak Republic

The critical values of the simultaneous test for the regression parameters, β , and the error's standard deviation, σ , of the linear regression model will be presented for small sample sizes, n , small numbers of explanatory variables, k , and for usual significance level.

Classification of Noisy Data with an Application to Breath Gas Analysis

Katarína Cimermanová

Institute of Measurement Science, Slovak Academy of Sciences, Bratislava, Slovak Republic

Classification of multidimensional data into one of two classes is an important problem. There are some classification methods which classify data to one of two classes, but in real live situations the measurement vectors are observed with additional uncertainty (noisy data). Solution to this problem is a robust formulation that stems from the Support Vector Machine method. The formulation is a convex optimization problem; in particular, it is an instance of the Second Order Cone Programming problem. An ellipsoidal uncertainty model is assumed in the robust formulation. It is derived from a worst case consideration and assumes only the



existence of the second order moments. The robust classification method is applied to breath gas analysis, where we classify volunteers to group of smokers and non-smokers.

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Empirical Likelihood Estimation in Interest Rate Diffusion Models

Lukáš Laffers^(a) and Marian Grendár^(b,c)

(a) Comenius University, Bratislava

(b) Matej Bel University, Banská Bystrica

(c) Institute of Measurement Science, Slovak Academy of Sciences, Bratislava
Slovak Republic

Empirical Likelihood (EL) with estimating equations is applied to estimate parameters of Vasicek model and Cox-Ingersoll-Ross model. For Vasicek Exponential Jumps model estimating equations based on conditional characteristic function can be utilized in EL framework, but they lead to unstable estimates.

Special Matrices in Time Series Forecasting

Martina Hančová

Institute of Mathematics, Faculty of Science, P.J. Safarik University, Košice, Slovakia

Time series analysis and forecasting resulting from linear regression models can be established on the theory of Hilbert spaces, which allows us to apply simultaneously concepts of geometry and linear algebra together with ideas belonging to mathematical analysis. Our contribution will present two examples of such important concepts, two special matrices: the Gram matrix and Schur complement.



Plug-In Problem in Discrete GNSS-PIM Algorithms and Nonsensitiveness Region for Test of Position Integrity

Jana Heckenbergerová^(a), Hana Boháčová^(b) and Jaroslav Marek^(c)

(a) Faculty of Electrical Engineering and Informatics, University of Pardubice

(b) Faculty of Economics and Administration, University of Pardubice

(c) Department of Mathematical Analysis and Applications of Mathematics, Palacky University in Olomouc, Czech Republic.

The aim of the contributed paper is to find a solution of plug-in problem in Discrete GNSS-PIM algorithms.

At first, known results in Discrete GNSS-PIM algorithms are introduced. Then it is necessary to estimate variance components of GNSS positions covariance matrix, which is in form $\Sigma = \sigma_1^2 V_1 + \sigma_2^2 V_2$, where V_1, V_2 are known positive semidefinite matrices. Estimation of variance components σ_1^2, σ_2^2 is based on a comparison of projection residual vectors size during movement on straight train track lines with different deflection from the x axis.

Plug-in problem must be solved, when covariance matrix Σ is unknown and we would like to use its estimation instead. If confidence area of variance components σ_1^2, σ_2^2 is a subset of nonsensitiveness region for test of GNSS position integrity in a gain alternative, than estimation of covariance matrix can be considered as precise enough to use in a Discrete GNSS-PIM algorithm instead of a real covariance matrix. Confidence area and nonsensitiveness region for test in a gain alternative are derived in this paper as a necessary part of the plug-in problem solution.

In the end of this contribution, numerical illustration of the plug-in problem and its solution is performed, where real data of the train track and the GNSS train position are used.

Calibration Intervals for Values of Concentration Based on Measurements of Voltage

Klára Hornišová

Institute of Measurement Science, Slovak Academy of Sciences, Bratislava, Slovakia

For calibration of measurements made on device with bias non-randomly changing in time the modification of nonparametric method of Gruet is utilized and its properties are assessed and compared with other approaches.



Locally Best Linear-Quadratic Unbiased Estimators of the Covariance Matrix Elements in a Special Heteroscedastic Regression Model

Gejza Wimmer

Masaryk University, Brno, Czech Republic,
Mathematical Institute, Slovak Academy of Sciences, Bratislava, Slovak Republic,
Matej Bel University, Banská Bystrica, Slovakia

Explicit formulas are given for the locally best linear-quadratic unbiased estimators of the covariance matrix elements in the regression model

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \quad i = 1, 2, \dots, n,$$

where $\varepsilon_i \sim N(0, \sigma^2(a + b|\beta_0 + \beta_1 x_i|)^2)$ are independent, σ^2, a, b known constants, $x_i \neq x_j$ for $i \neq j$, $n \geq 4$. The variances of the derived estimators are also given and investigated in a special case of increasing number of measuring points x_i .

Comparison of Different Confidence Regions for Regression Parameter in Linear Mixed Model

Gejza Wimmer Jr.

Mathematical Institute, Slovak Academy of Sciences, Bratislava, Slovak Republic

Suppose that the response variable is observed on several subjects in time. This kind of data is known as longitudinal data and linear mixed model (LMM) is often used to analyze them. The regression parameters of the model (fixed effects) describe the common behavior of all observed subjects. In the presented simulation study we compare the coverage probabilities of some different confidence regions for regression parameters of LMM provided that errors satisfy AR(1) process in the case of different number of measurements on various number of subjects.

Orthogonal Decompositions in Growth Curve Models

Ivan Žežula and Daniel Klein

Institute of mathematics, Šafárik University, Košice, Slovak Republic

Usefulness of orthogonal decompositions in the basic and extended growth curve model will be discussed. Our results will be compared with those of Ye & Wang.



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List of Participants

Fikri Akdeniz

Department of Statistics
Faculty of Arts and Sciences
Çukurova University
Adana, Turkey
fikriakdeniz@gmail.com

Esra Akdeniz Duran

Department of Statistics
Faculty of Arts and Sciences
Gazi University
Ankara, Turkey
esraakdeniz@gmail.com

Barbora Arendacká

Institute of Measurement Science
Slovak Academy of Sciences
Bratislava, Slovak Republic
barendacka@gmail.com

Philip Victor Bertrand

Consultant Statistician
Solihull, UK
pbbg12972@blueyonder.co.uk

Hana Boháčová

Institute of Mathematics
Faculty of Economics and
Administration
University of Pardubice
Pardubice, Czech Republic
hana.bohacova@upce.cz

Martina Chvosteková

Institute of Measurement Science
Slovak Academy of Sciences
Bratislava, Slovak Republic
chvosta@gmail.com

Katarína Cimermanová

Institute of Measurement Science
Slovak Academy of Sciences
Bratislava, Slovak Republic
katarina.cimermanova@gmail.com

Hilmar Drygas

FB 17 (Math)
University Kassel
Körle-Empfeshausen, Germany
hilmar.drygas@onlinehome.de

Dário Ferreira

Department of Mathematics
University of Beira Interior
Covilhã, Portugal
dario@mat.ubi.pt

Sandra Saraiva Ferreira

Department of Mathematics
University of Beira Interior
Covilhã, Portugal
sandra@mat.ubi.pt

Miroslav Fiedler

Institute of Computer Science
Academy of Sciences
Prague, Czech Republic
fiedler@cs.cas.cz

Jolanta Grala-Michalak

Adam Mickiewicz University
Poznań, Poland
grala@amu.edu.pl

Marian Grendár

Matej Bel University
Banská Bystrica, Slovak Republic
marian.grendar@savba.sk

Karl Gustafson

Department of Mathematics
University of Colorado at Boulder
Boulder, USA
Karl.Gustafson@Colorado.EDU

Frank Jerry Hall

Department of Mathematics and
Statistics
Georgia State University
Atlanta, USA
matfjh@langate.gsu.edu

Martina Hančová

Institute of Mathematics
Šafárik University
Košice, Slovak Republic
martina.hancova@upjs.sk

Radoslav Harman

Department of Applied
Mathematics and Statistics
Faculty of Mathematics,
Physics and Informatics
Comenius University
Bratislava, Slovak Republic
harman@fmph.uniba.sk

Jan Hauke

Institute of Socio-Economic
Geography and Spatial Management
Adam Mickiewicz University
Poznań, Poland
jhauke@amu.edu.pl

Jana Heckenbergerová

Faculty of Electrical Engineering
and Informatics
University of Pardubice
Pardubice, Czech Republic
jana.heckenbergerova@upce.cz

Klára Hornišová

Institute of Measurement Science
Slovak Academy of Sciences
Bratislava, Slovak Republic
Klara.Hornisova@savba.sk

Karel Hron

Department of Mathematical
Analysis and Applications of
Mathematics
Faculty of Science
Palacky University
Olomouc, Czech Republic
hronk@seznam.cz

Jeffrey Joseph Hunter

Auckland University
of Technology
Auckland, New Zealand
jeffrey.hunter@aut.ac.nz

Stanislav Katina

Department of Applied
Mathematics and Statistics
Faculty of Mathematics,
Physics and Informatics
Comenius University
Bratislava, Slovak Republic
katina@fmph.uniba.sk

Mike Kenward

London School of Hygiene &
Tropical Medicine
University of London
London, UK
Mike.Kenward@lshtm.ac.uk

Daniel Klein

Institute of Mathematics
Šafárik University
Košice, Slovak Republic
daniel.klein@upjs.sk

Thomas Klein

Zentrum Mathematik
Technische Universität München
Garching, Germany
tpklein@ma.tum.de



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List of Participants

Tõnu Kollo

Institute of Mathematical Statistics
University of Tartu
Tartu, Estonia
tonu.kollo@ut.ee

Lubomír Kubáček

Department of Mathematical
Analysis and Applications of
Mathematics
Faculty of Science
Palacky University
Olomouc, Czech Republic
kubacekl@inf.upol.cz

Lynn Roy LaMotte

Louisiana State University
Health Sciences Center
New Orleans, USA
llamot@lsuhsc.edu

Natalja Lepik

Institute of Mathematical Statistics
University of Tartu
Tartu, Estonia
natalja.lepik@ut.ee

Yonghui Liu

Department of Applied
Mathematics
Shanghai Finance University
Shanghai, China
liuyh.shfc@gmail.com

Peter Maličský

Faculty of Natural Sciences
Matej Bell University
Banská Bystrica, Slovak Republic
maličky@fpv.umb.sk

Ivo Marek

Faculty of Civil Engineering
Czech University of Technology
Prague, Czech Republic
marek@ms.mff.cuni.cz

Märt Möls

Institute of Mathematical Statistics
University of Tartu
Tartu, Estonia
martm@ut.ee

Célia Nunes

Department of Mathematics
University of Beira Interior
Covilhã, Portugal
celia@mat.ubi.pt

Martin Ohlson

Linköping University
Linköping, Sweden
martin.ohlson@liu.se

Andrej Pázman

Department of Applied
Mathematics and Statistics
Faculty of Mathematics,
Physics and Informatics
Comenius University
Bratislava, Slovak Republic
pazman@fmph.uniba.sk

Friedrich Pukelsheim

Universität Augsburg
Augsburg, Germany
Pukelsheim@Math.Uni-Augsburg.De

Simo Puntanen

Department of Mathematics and
Statistics
University of Tampere
Tampere, Finland
Simo.Puntanen@uta.fi

František Rublík

Institute of Measurement Science
Slovak Academy of Sciences
Bratislava, Slovak Republic
umerrubl@savba.sk

Burkhard Schaffrin

School of Earth Sciences
The Ohio State University
Columbus, USA
aschaffrin@earthlink.net

Alastair John Scott

Department of Statistics
University of Auckland
Auckland, New Zealand
scott@stat.auckland.ac.nz

Anne Selart

Institute of Mathematical Statistics
University of Tartu
Tartu, Estonia
anne.selart@ut.ee

Carlos Ferreira Sousa

Universidade do Algarve
Faro, Portugal
cfsousa@ualg.pt

Svorad Štolc

Institute of Measurement Science
Slovak Academy of Sciences
Bratislava, Slovak Republic
S.Stolc@savba.sk

George P.H. Styan

Department of Mathematics and
Statistics
McGill University
Montréal, Canada
styan@math.mcgill.ca

Gerald E. Subak-Sharpe

City College New York
New York, USA
gssharp@ix.netcom.com

Imbi Traat

Institute of Mathematical Statistics
University of Tartu
Tartu, Estonia
imbi.traat@ut.ee

Júlia Volaufová

Louisiana State University
Health Sciences Center
New Orleans, USA
jvolau@lsuhsc.edu

Dietrich von Rosen

Department of Energy &
Technology
Swedish University of
Agricultural Sciences
Uppsala, Sweden
Dietrich.von.Rosen@et.slu.se

Tatjana von Rosen

Department of Statistics
Stockholm University
Stockholm, Sweden
Tatjana.vonRosen@stat.su.se

Hans Joachim Werner

Department of Statistics
Faculty of Law & Economics,
University of Bonn
Bonn, Germany
hjw.de@uni-bonn.de

Gejza Wimmer

Masaryk University Brno
Brno, Czech Republic
wimmer@mat.savba.sk



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List of Participants

Gejza Wimmer Jr.

Mathematical Institute
Slovak Academy of Sciences
Bratislava, Slovak Republic
dzibo7@gmail.com

Viktor Witkovský

Institute of Measurement Science
Slovak Academy of Sciences
Bratislava, Slovak Republic
witkovsky@savba.sk

Ivan Žežula

Institute of Mathematics
Šafárik University
Košice, Slovak Republic
ivan.zezula@upjs.sk

Roman Zmysłony

Faculty of Mathematics
Computer Science and
Econometrics
University of Zielona Góra
Zielona Góra, Poland
r.zmyslony@wmie.uz.zgora.pl



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