Power-law decay vs. fractal complexity of EEG

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Abstract. *The paper deals with possible connection between spectrum power-law decay and correlation dimension estimated for electroencephalogram (EEG). About 2300 EEG data recorded during relaxed wakefulness were analysed. The whole spectrum of EEG was studied and power-law decay of about 2.28 prevailing over the exponential falling off was established. The mean value of correlation dimension of 4.35 was estimated. The discriminating power of both quantities seems to be comparable.*

Keywords: Electroencephalography; EEG; Relaxation; Spectral power; Spectral decay; Correlation dimension

1. Introduction

In the last decades the growing need for a better understanding of large-scale brain dynamics has stimulated the expansion of research in this area. In contrast to the linear Fourier analysis based description, there are many arguments for neuronal dynamics to be considered to behave in a non-linear manner. This fact has prompted the development of new analysis techniques, often referred to as *non-linear methods*.

Among the various non-linear methods available for experimental data, the calculation of the fractal complexity has probably received the widest attention. It has been mathematically established that, if we can measure any single variable of a dynamical system with sufficient accuracy, then it is possible to reconstruct a state portrait, topologically equivalent to the attractor of the original system. The complexity of an attractor of the reconstructed behavior may provide important information about the system. The most popular tool to assess this complexity is the correlation dimension defined as:

$$D_2 = \lim_{\epsilon \to 0} \frac{\ln \sum_{i=1}^{N(\epsilon)} p_i^2}{\ln \epsilon} = \lim_{\epsilon \to 0} \frac{\ln C_2(\epsilon)}{\ln \epsilon}$$

where $N(\varepsilon)$ is the total number of hyper-cubes of side length ε, which cover the attractor, and p_i is the probability of finding a point in the hypercube *i*. As Grassberger and Procaccia noticed [1] $C_2(\varepsilon)$ is approximately equal to the probability that the distance between two points of the attractor is less then ε:

$$C_2(\epsilon) = \frac{2}{N(N-1)} \sum_{i}^{N} \sum_{j>i}^{N} \Theta(\epsilon - \|\mathrm{x}_m(i) - \mathrm{x}_m(j)\|)$$

where theta is the Heaviside step function, and ||.|| usually represents the maximum norm.

In order to estimate the correlation dimension, we plot ln $C_2(\varepsilon)$ as a function of $\ln \varepsilon$ and follow the slope $\nu(\varepsilon)$ of the obtained curve. This slope . is called a correlation exponent, and the limit of it for vanishing ε represents the value of correlation dimension.

The relative simplicity of the method of Grassberger and Procaccia (GP-method) ended in numerous applications. EEG has been a matter of interest for the dimension computation since the first D_2 estimates for sleep cycles were made by Babloyantz et al. [2]. This attempt was inspired by the chaos hypothesis, i.e. it was assumed that the EEG could be described by a deterministic chaotic system and therefore the corresponding attractor could be characterized by the fractal dimension.

Already from the beginning these efforts have been questioned for several reasons: If the existence of an attractor is assumed, then for a reliable dimension estimate the time series has to be long enough and fulfil requirements such as stationarity and a reasonable signal to noise ratio. In fact, EEG seems to be a mixture of noise, some cyclic processes and random fractal signals. Each part of such a composition itself has been frequently reported to fool the GP-algorithm as it came to light shortly after its first applications.

Despite the criticism, measures used for identifying low-dimensional chaotic systems, such as the correlation dimension, continue to be used for studying the EEG signals. For instance, some authors reported decreasing values of D_2 with deepening of the level of sleep or the level of release [2,3,4].

Now let us return to the most commonly used technique for analyzing EEG that is Fourier analysis. Regarding spectral properties of different types of data the next statements are generally accepted:

Stochastic behaviour: the power spectrum decays via a power law $P(f)\sim 1/f^{\,\gamma}$ (γ can be obtained as the slope of linear part when plotted on a log-log scale) [5].

Periodic or quasi-periodic behaviour: the power spectrum consists of discrete spikes corresponding to distinct frequencies.

Chaotic behaviour: the power spectrum falls exponentially at high frequencies [6]. Exponential decay of power spectrum is a decay of the form $P(f)\sim a.e^{-bf}$, where a and the exponent b are positive constants. (This region is linear when plotted on a log-linear scale.)

The above summary shows that the falloff of the power spectrum helps us to answer the question, whether the observed erratic behaviour is essentially deterministic or stochastic.

As the power of EEG spectrum is supposed to decrease polynomially, let us mention a result that advocate the use of the order of the polynomial falloff as a tool for dimension estimate. In [7] the authors found that for a stochastic time series with a $1/f^{\,\gamma}$ power spectrum, the numerical estimate of correlation dimension is a small finite value $D=2/(\gamma-1)$ for $1<\gamma<3$, when analysed by the GP-algorithm.

In the case of EEG, Pereda et al. [4] investigated a correlation between the spectral exponent and D_2. They selected the frequency range of 3–30 Hz and found, that EEG exhibits random fractal structure with $1/f^{\,\gamma}$ spectrum and a negative linear correlation between D_2 and γ is present in all states except during slow wave sleep. In [8] the authors estimated the dimension of the sleep EEG in a range from about 6.78 to about 9.82. The corresponding decay rate (computed for frequencies less then alpha activity) was in a range *0,98-2.18*.

Our aim is to verify declarations about the relation between D_2 and spectral decay exponent. At first we checked the presence of exponential or power-law decay in the power spectra of EEG. As there seems to be no consensus regarding the choice of the regions of power-law decay in the literature, we studied the whole spectrum of EEG to find the sections of clear power-law decay.

Thereafter we looked for regions that lead to highest linear correlation and mutual information between both characteristics.

Finally, we thought over the discriminating power of spectral decay γ and correlation dimension, as in the case of equivalence, the use of γ can be recommended as more effective due to its lower computational cost.

2. Subject and Methods

Eight healthy volunteers (3 females and 5 males) took part in EEG recording. Participants ranged in age from 24 to 39 years, with a mean of 25.5 years, s.d. 5.1 yrs.. They attended 2 measurements per each of 25 days. Data of 3-minute length were recorded. During recording subjects were lying in a darkened, electrically shielded room. They were instructed to keep their eyes closed and relax both physically and mentally.

8-channel EEG system with scalp-electrode impedances kept below 5 K Ohms was used for data recording. From the 8 signals (6 active electrodes and 2 reference electrodes) six difference signals were derived by. A digital high pass FIR filter with cut-off at 0.75 Hz, with the width of 3000 data points, and with a Blackman window was utilized.

For the purpose of this study, about 2300 electroencephalograms were analyzed. The EEG measures were computed from 3-minute epochs, which were digitized at 500 Hz. Following digital filtering the first and the last 1500 points were omitted and 87000 data point EEGs remained.

3. Results

In order to calculate the correlation dimension, the data were embedded to *m*-dimensional space (*m*=1, 2, …, 20). Following the proposal of Takens [9] *m*-dimensional vectors built-up from delay coordinates were used for the reconstruction. The vectors were constructed with a time lag $\tau = 10$, which corresponds to 5 ms. This value was chosen according to the first minimum of mutual information between the original signal and its shifted versions, meaning, that their independence is maximized [10]. Then D_2 was calculated using the GP–algorithm.

The resultant estimates of D_2 “saturate”, i.e. they approach a constant value as embedding dimensions increase above *m*=5. After this manner a significant indication of relatively low values of correlation dimension (between 3 and 6) with the mean of 4.35 was found.

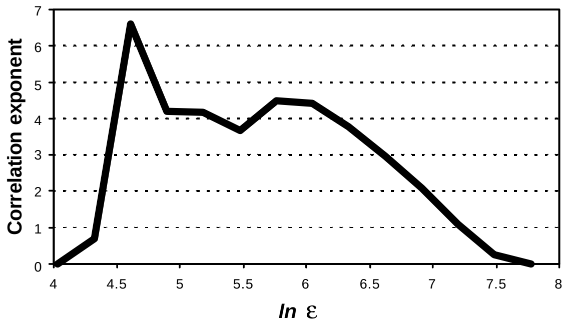

Fig. 1. Typical graph of correlation dimension estimation for EEG. *N*=87000, *m*=6. Plateau at about 4.1 is apparent.

Power spectrum was computed using standard FFT with frequency step of 0.029 Hz and variance reduction factor of 10. Spectrum presented in *lin-log* graph shows apparent peak of about 10 Hz corresponding to brain alpha frequency. Similar phenomenon is present in *log-log* graph as well. However, in this case frequencies before and after peak fall linearly in contrast to *lin-log* graph. Power law decay γ was computed as a slope of linear regression applied to the power spectrum in log-log graph. Because *log* function condense data points in higher frequencies, data should be recalculated with respect to homogenous distribution along axis *x*. This can be done using linear re-sampling of interpolated data in *log-log* graph.

As a result, power-law decay of about 2.28 prevailing over the exponential falling off was established. Fig.2 shows a typical graph for decay estimation. Fig. 3. illustrates, how successful is power-law model in comparison to exponential model.

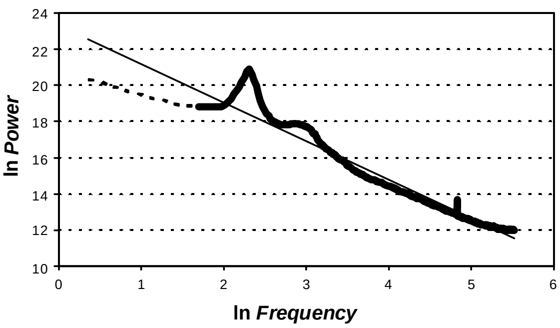

Fig. 2. Typical graph of spectral decay estimation for EEG. Decay of about 2.28 is visible.

To evaluate the relationship between the correlation dimension estimates and the spectral decay we computed linear correlation and mutual information of both measures. As fig. 3. shows we found maximum information (linear correlation of 0.73) when spectral decay from the whole spectrum (in our case about 5 Hz to 250 Hz) is taken. Fig. 4. illustrates, that both D_2 and γ behave in a similar manner and they probably reflect the same features of EEG data.

4. Discussion and conclusions

As our results confirm, to a great extent the dimension estimate by GP-algorithm reflects some spectral features of signal.

Despite this fact, the correlation dimension may remain usable as one of invariants of underlying system. If computed with highest caution the estimated value is expected to provide a valuable relative, generic measure of the dynamical complexity of a signal. But it is questionable if the correlation dimension can be more powerful as the spectral decay. Since power spectra can be easily calculated with standard algorithms, estimating spectral characteristics is advantageous in comparison with time consuming algorithms to compute the correlation dimension.

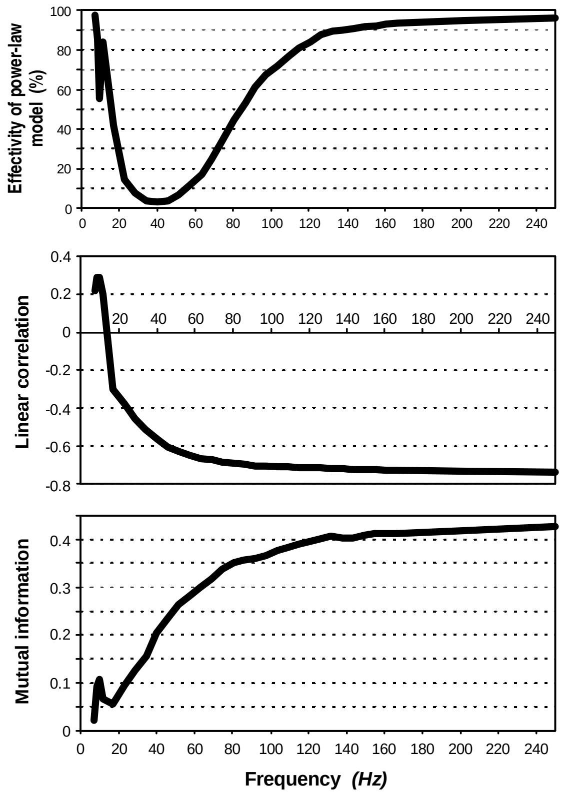

Fig. 3. a) Efficiency of power-law model in contrast to exponential model when power spectra from fixed 5 Hz to ascending right limit of the spectral range is investigated.
b) Linear correlation between D_2 and γ.
c) Mutual information between D_2 and γ.

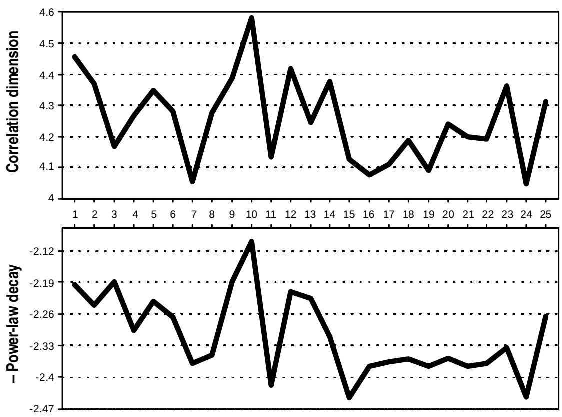

Fig. 4. Averages of spectral decay and correlation dimension values in the course of 25 days. Computed from 8 subjects EEGs.

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