EEG: Spectral Characteristics vs. Correlation Dimension

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INTRODUCTION
The poster shows our results regarding the role of spectral properties (autocorrelations and exponential or power-law decay) in a process of correlation dimension estimation for EEG.

CORRELATION DIMENSION
is a measure which quantifies the complexity of the system:

\[ D_2 = \lim_{\epsilon \to 0} \frac{\sum_{i=1}^{N(x)} \frac{N_i(x)}{N}}{\ln \epsilon} \]

where \( N_i(x) \) is the total number of hypercubes of side length \( \epsilon \) which cover the attractor, and \( p_i \) is the probability of finding a point in the hypercube at \( x \). In order to estimate the correlation dimension, we have to plot \( \ln C(x) \) as a function of \( \ln \epsilon \) and follow the slope of the obtained curve. This slope \( \ln C(x) \) is called correlation exponent, and the limit of it for vanishing \( x \) represents the value of correlation dimension. The most widely used algorithm to estimate this quantity is that due to Grassberger and Procaccia (GP) [1].

AUTOCORRELATION vs. CORRELATION DIMENSION

Theller [2] has shown that for data sets with long autocorrelation time the application of standard correlation dimension estimation algorithms can yield spurious. The measure of autocorrelation and exponential decay indicate, that the plateaus in dimension plots are spurious.

Although application of correlation dimension analysis on EEG gives interesting answers for low-dimensional deterministic dynamics, it can yield different meaningless answers.

CORRELATION DIMENSION ESTIMATE FROM EEG DATA

In real systems like EEG the chaos is very difficult to prove or exclude. The scepticism against finite dimension estimates is understandable. It is hard to believe that a complicated system as the brain, which is continually interacting with other complex systems, should manifest as deterministic low-dimensional dynamics. Preeminently, it is a manifestation of a mixture of noise, some cyclic processes and random fractal signals. Each part of such a composition itself is frequently reported to fool the algorithms used to detect chaotic dynamics. Therefore, the correlation dimension estimates should be interpreted with extreme caution.

Unfortunately, in real systems the determinism is very difficult to distinguish from power-law, fractal noise. High-frequency power-law behaviour in power spectra is, in fact a general property of noisy data. We looked for any region of exponential decay prior to final power-law decay. But in more than 90% of EEG data the power-law spectral decay was established. Then the hypothesis of presence of scale-invariant fractal structures in brain dynamics should be preferred rather than the hypothesis of deterministic chaos.

Although it becomes clear that claims for evidence of low-dimensional dynamics in EEG are probably incredible, a large number of published results indicate that dimension analysis of EEG time series yields reasonable and useful information. E.g. our numerical experiments suggest that the spectral decay rate \( \alpha \) (and alike the measure of autocorrelation) is inversely proportional to the dimension estimates and altogether show definite changes in long-term relaxation process. It seems that the finite dimension estimates can not be longer claimed to approve deterministic low-dimensional dynamics underlying the EEG. Nevertheless the correlation dimension remains usable as one of invariants of underlying system. For a great extent the dimension estimate by GP-algorithm only reflects some spectral features of signal. Therefore, it is questionable if the correlation dimension estimate can be more powerful as the majority of other characteristics.

RESULTS AND CONCLUSIONS

At average, correlation dimension plot for relaxed EEG shows plateau about 3.7–3.8. As the hypothesis of low-dimensional chaotic behaviour must be consistent with the results of spectral analysis we tested for the presence of autocorrelations and exponential or power-law decay in the power spectra.

To avoid spurious dimension estimates, Theller recommends to take much more data than the characteristic autocorrelation time \( \tau \), and omit pairs of points closer in time than \( \tau \).

DECLAY OF POWER SPECTRA vs. CORRELATION DIMENSION
It is generally accepted that:

Power spectra from deterministic chaotic systems decay exponentially at high frequencies (exponential decay of power spectrum is a decay of the form \( P(f) \sim f^{-\alpha} \), where \( \alpha \) and \( f \) are positive constants). Since noise is always present in real systems, one can, in case of real chaotic systems, observe only a finite region of exponential decay. Then the spectrum settles into the power-law decay characteristic of noise. As [3] shows, the observation of a finite region of exponential decay is a sufficient condition for the system to be essentially deterministic.

Power spectra from random or stochastic systems decay via a power-law (i.e. power spectrum decreases as \( f^{-\alpha} \) with increasing frequency \( f \)).

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REFERENCES

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